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Machine Learning Algorithm

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Machine Learning Algorithms

- **Machine Learning algorithm**: a procedure in developing computer programs that improve their performance with “experience.”
- Types of machine learning: Naïve Bayes, Linear Regression, Support Vector Machine, Decision Tree, Neural Network, Deep Learning, and so on.
- Learning all algorithms are over the scope of this summer boot camp. We will focus on Naïve Bayes as an example of Machine Learning.

Outline

- **Basic Probability and Notation**
- Bayes Law and Naive Bayes Classification
- Smoothing
- Class Prior Probabilities
- Naive Bayes Classification
- Summary

Crash Course in Basic Probability

Discrete Random Variable

- A is a discrete random variable if:
  - A describes an event with a finite number of possible outcomes (discrete vs continuous)
  - A describes and event whose outcomes have some degree of uncertainty (random vs. pre-determined)

Discrete Random Variables

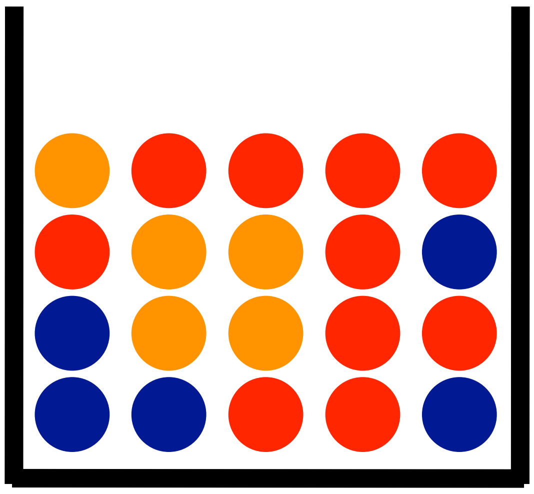
Examples

- A = the outcome of a coin-flip
  - outcomes: heads, tails
- A = it will rain tomorrow
  - outcomes: rain, no rain
- A = you have the flu
  - outcomes: flu, no flu

Discrete Random Variables

Examples

- A = the color of a ball pulled out from this bag
  - outcomes: RED, BLUE, ORANGE

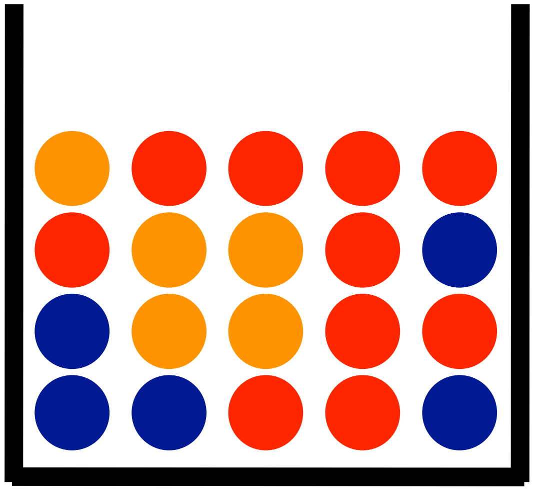


Probabilities

- Let P(A=X) denote the probability that the outcome of event A equals X
- For simplicity, we often express P(A=X) as P(X)
- Ex: P(RAIN), P(NO RAIN), P(FLU), P(NO FLU), ...

Probability Distribution

- A probability distribution gives the probability of each possible outcome of a random variable
- P(RED) = probability of pulling out a red ball
- P(BLUE) = probability of pulling out a blue ball
- P(ORANGE) = probability of pulling out an orange ball



Probability Distribution

- For it to be a probability distribution, two conditions must be satisfied:
  - the probability assigned to each possible outcome must be between 0 and 1 (inclusive)
  - the sum of probabilities assigned to all outcomes must equal 1

$$0 \le P(RED) \le 1$$

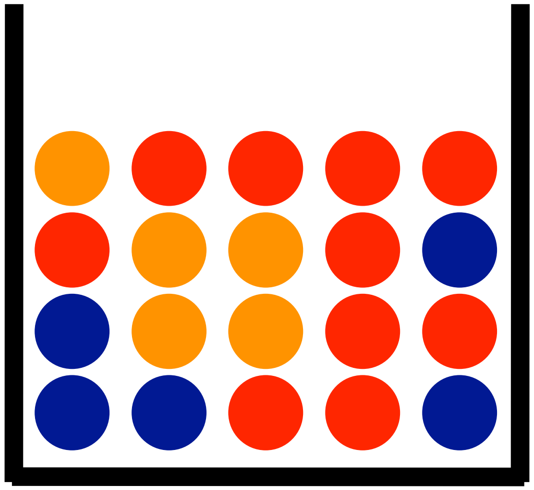
$$0 \le P(BLUE) \le 1$$

$$0 \le P(ORANGE) \le 1$$

$$P(RED) + P(BLUE) + P(ORANGE) = 1$$

Probability Distribution
Estimation

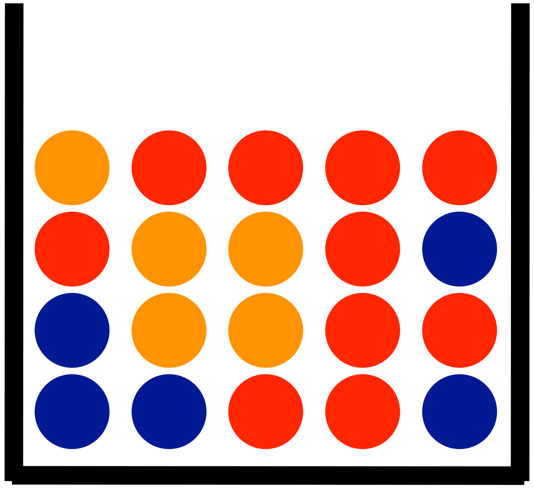
- Let's estimate these probabilities based on what we know about the contents of the bag
- P(RED) = ?
- P(BLUE) = ?
- P(ORANGE) = ?



Probability Distribution

estimation

- Let's estimate these probabilities based on what we know about the contents of the bag
- P(RED) = 10/20 = 0.5
- P(BLUE) = 5/20 = 0.25
- P(ORANGE) = 5/20 = 0.25
- P(RED) + P(BLUE) + P(ORANGE) = 1.0



Probability Distribution

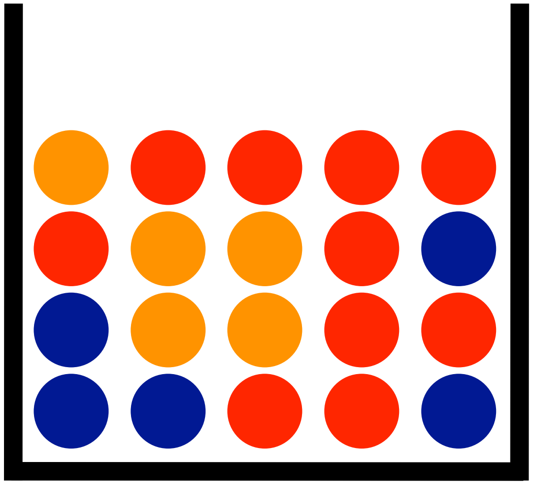
assigning probabilities to outcomes

- Given a probability distribution, we can assign probabilities to different outcomes
- I reach into the bag and pull out an orange ball. What is the probability of that happening?
- I reach into the bag and pull out two balls: one red, one blue. What is the probability of that happening?
- What about three orange balls?

P(RED) =

P(BLUE) =

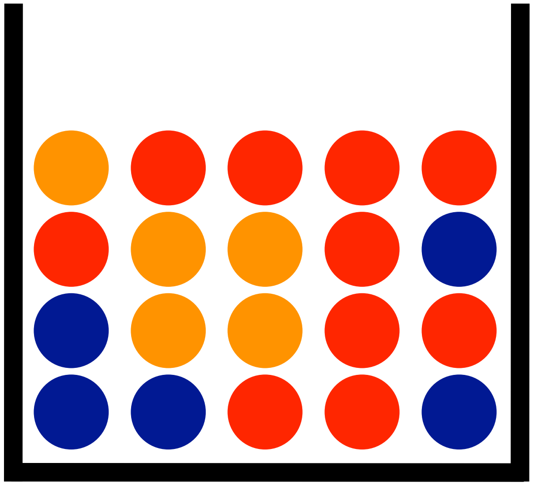
P(ORANGE) =



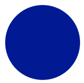
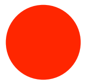
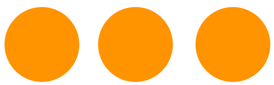
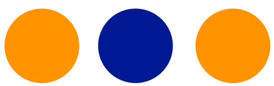
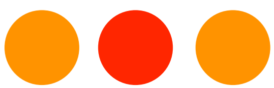
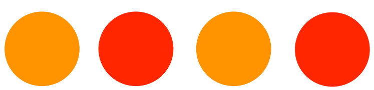
What can we do with a probability distribution?

- If we assume that each outcome is independent of previous outcomes, then the probability of a <u>sequence</u> of outcomes is calculated by <u>multiplying</u> the individual probabilities
- Note: we're assuming that when you take out a ball, you put it back in the bag before taking another

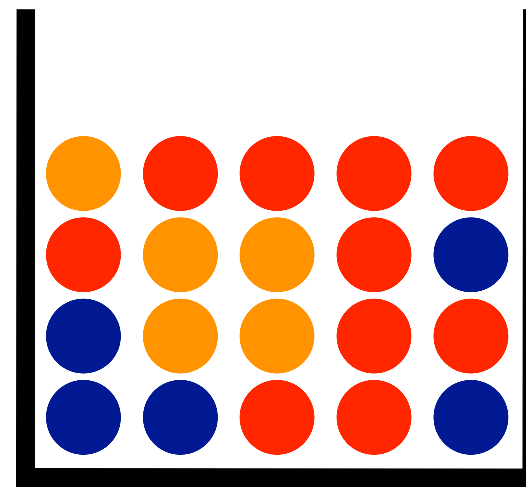
P(RED) = 0.5
P(BLUE) = 0.25
P(ORANGE) = 0.25



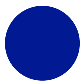
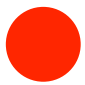
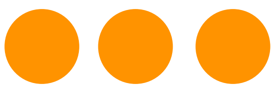
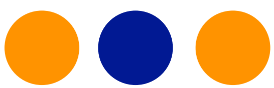
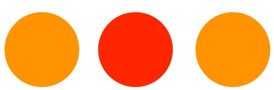
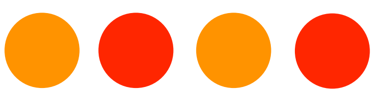
What can we do with a probability distribution?

- P() = ??
- P() = ??
- P() = ??
- P() = ??
- P() = ??
- P() = ??

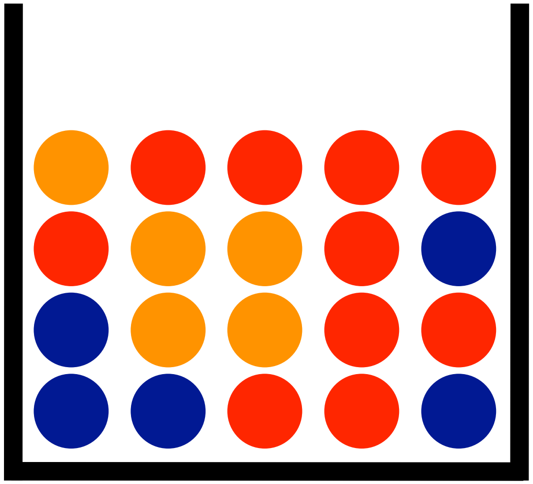
P(RED) = 0.5
P(BLUE) = 0.25
P(ORANGE) = 0.25



What can we do with a probability distribution?

- P() =
- P() =
- P() =
- P() =
- P() =
- P() =

P(RED) = 0.5
P(BLUE) = 0.25
P(ORANGE) = 0.25



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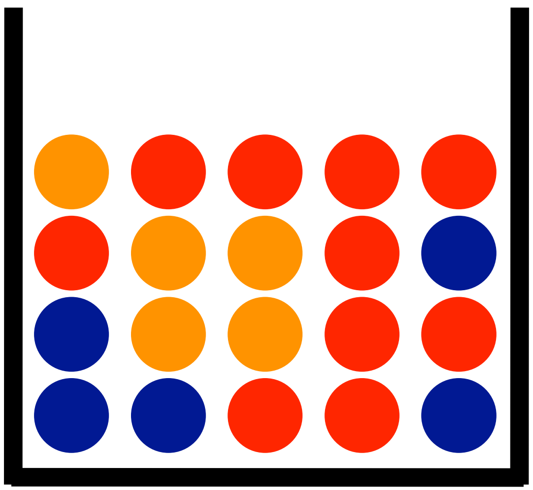
Joint and Conditional Probability

- P(A,B): the probability that event A and event B both occur
- P(A|B): the probability of event A occurring given prior knowledge that event B occurred

Conditional Probability

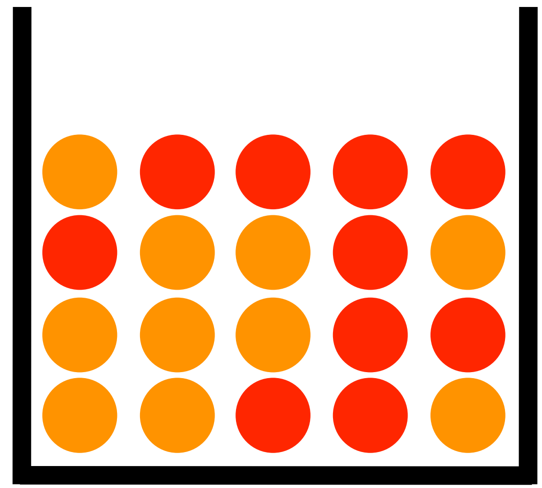
P(RED) = 0.50
P(BLUE) = 0.25
P(ORANGE) = 0.25



A

- P(| A) = 0.25
- P(| A) = 0.5
- P(| A) = 0.016

P(RED) = 0.50
P(BLUE) = 0.00
P(ORANGE) = 0.50



B

- P(| B) = 0.00
- P(| B) = 0.25
- P(| B) = 0.00

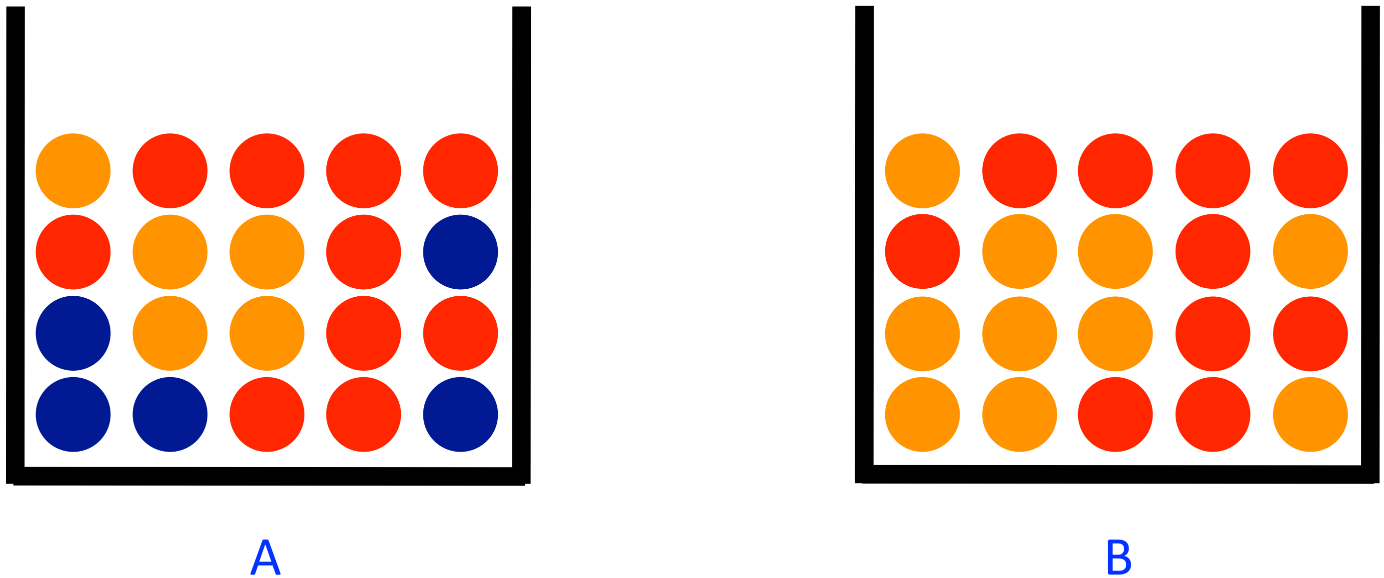
Chain Rule

- $P(A, B) = P(A|B) \times P(B)$
- Example:
  - probability that it will rain today (B) and tomorrow (A)
  - probability that it will rain today (B)
  - probability that it will rain tomorrow (A) given that it will rain today (B)

Independence

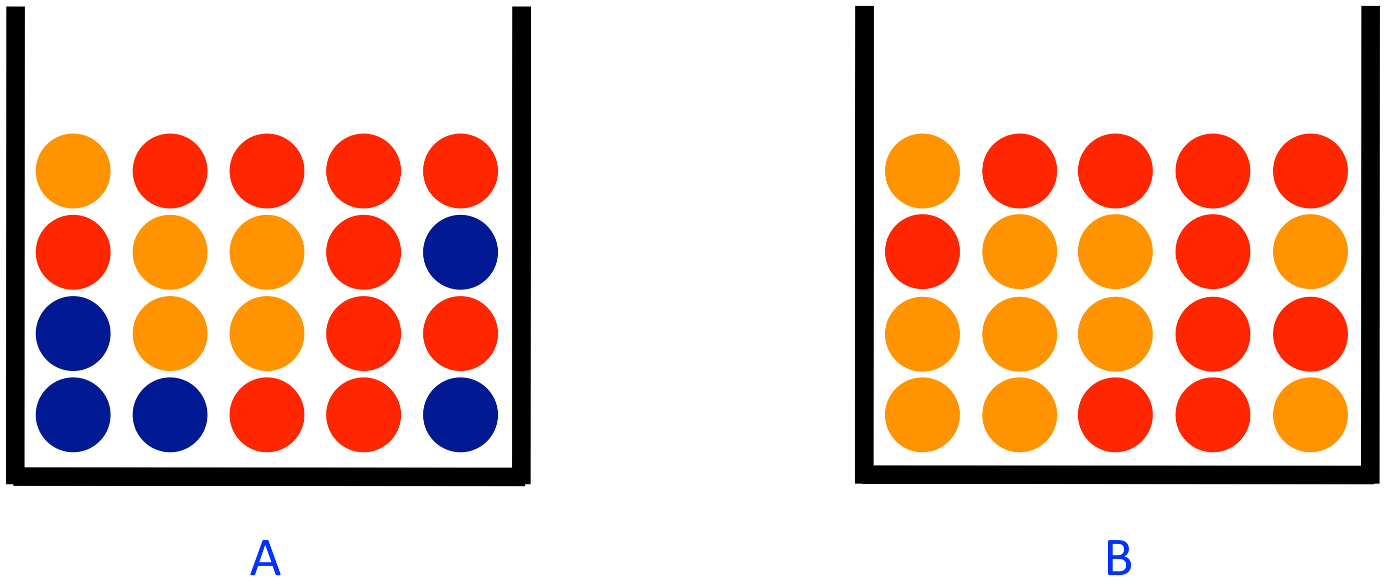
- $P(A, B) = P(A|B) \times P(B) = P(A) \times P(B)$
- Example:
  - probability that it will rain today (B) <u>and</u> tomorrow (A)
  - probability that it will rain today (B)
  - probability that it will rain tomorrow (A) given that it will rain today (B)
  - probability that it will rain tomorrow (A)

Independence



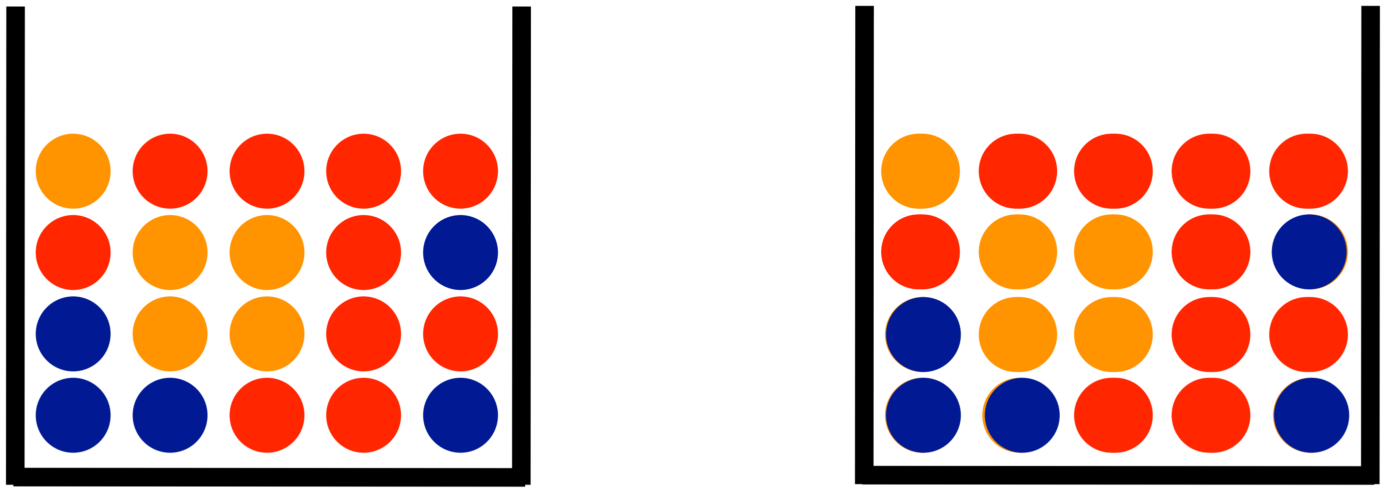
P(● | A) ?= P(●)

Independence



$$P(\bullet \mid A) > P(\bullet)$$

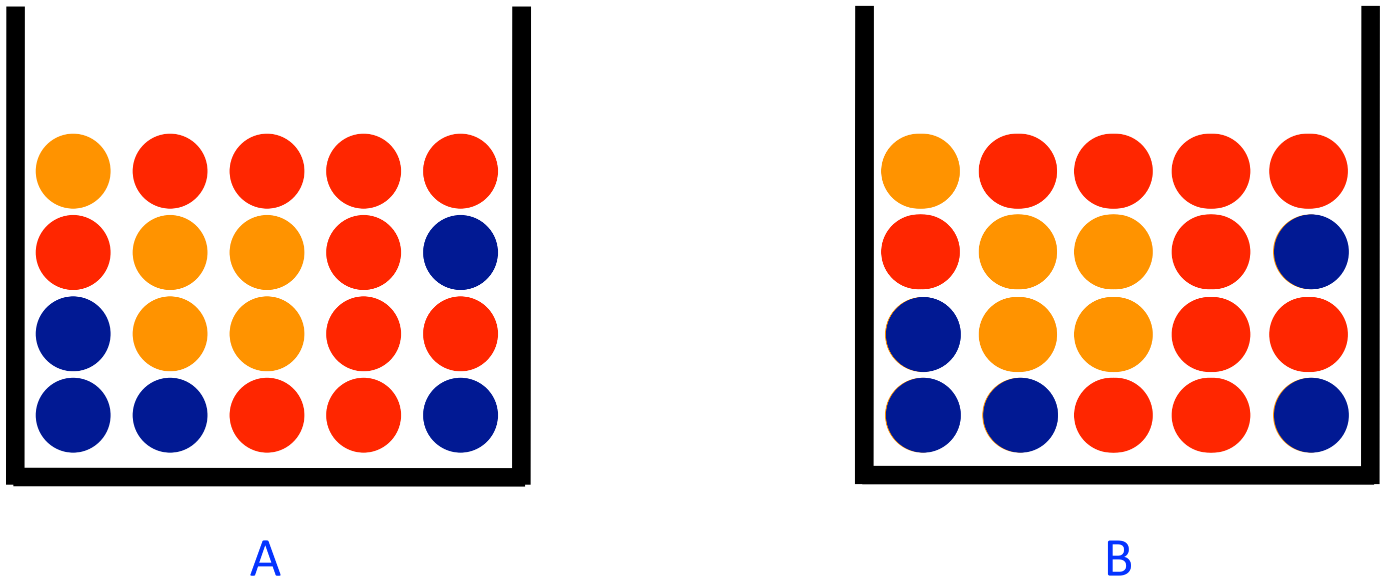
Independence



A B

P(● | A) ?= P(●)

Independence



$$P(\bullet \mid A) = P(\bullet)$$

Outline

- Basic Probability and Notation
- **Bayes Law and Naive Bayes Classification**
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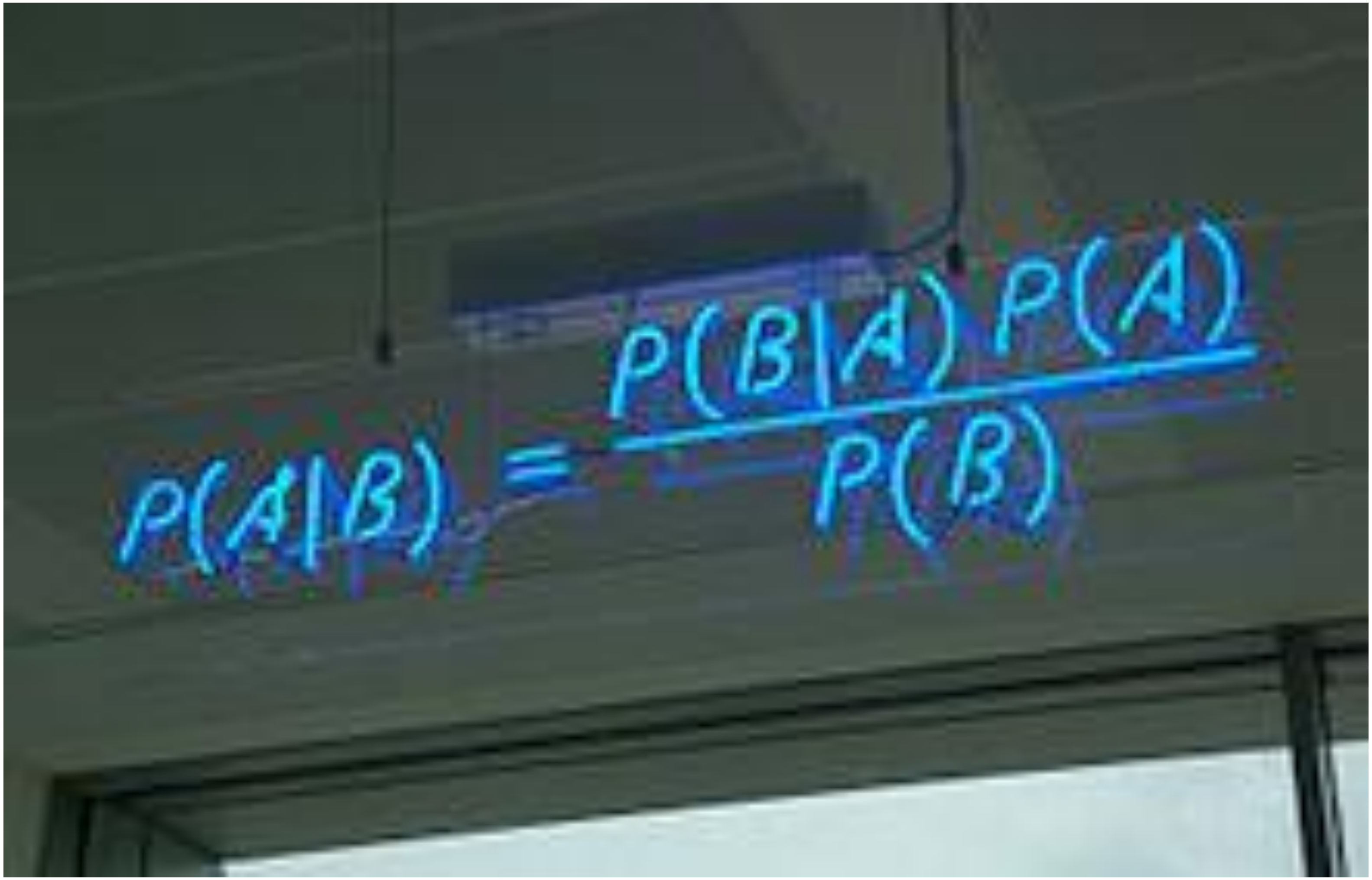
Bayes' Law



Bayes' Law

$$P(A|B) = \frac{P(B|A) \times P(A)}{P(B)}$$

Derivation of Bayes' Law

$$P(A,B) = P(A,B)$$

Always true!

$$P(A|B) \times P(B) = P(B|A) \times P(A)$$

Chain Rule!

$$P(A|B) = \frac{P(B|A) \times P(A)}{P(B)}$$

Divide both sides by P(B)!

Naive Bayes Classification

example: positive/negative movie reviews

Bayes Rule

$$P(A|B) = \frac{P(B|A) \times P(A)}{P(B)}$$

Confidence of POS prediction given instance D

$$P(POS|D) = \frac{P(D|POS) \times P(POS)}{P(D)}$$

Confidence of NEG prediction given instance D

$$P(NEG|D) = \frac{P(D|NEG) \times P(NEG)}{P(D)}$$

Naive Bayes Classification

example: positive/negative movie reviews

- Given instance D, predict positive (POS) if:

$$P(POS|D) \geq P(NEG|D)$$

- Otherwise, predict negative (NEG)

Naive Bayes Classification

example: positive/negative movie reviews

- Given instance D, predict positive (POS) if:

$$\frac{P(D|POS) \times P(POS)}{P(D)} \geq \frac{P(D|NEG) \times P(NEG)}{P(D)}$$

$$P(POS|D) \qquad\qquad P(NEG|D)$$

- Otherwise, predict negative (NEG)

Naive Bayes Classification

example: positive/negative movie reviews

- Given instance D, predict positive (POS) if:

$$\frac{P(D|POS) \times P(POS)}{P(D)} \geq \frac{P(D|NEG) \times P(NEG)}{P(D)}$$

Are these necessary?

- Otherwise, predict negative (NEG)

Naive Bayes Classification

example: positive/negative movie reviews

- Given instance D, predict positive (POS) if:

$$P(D|POS) \times P(POS) \geq P(D|NEG) \times P(NEG)$$

- Otherwise, predict negative (NEG)

Naive Bayes Classification

example: positive/negative movie reviews

- Our next goal is to estimate these parameters from the training data!
- P(NEG) = ??
- P(POS) = ??
- P(D|NEG) = ??
- P(D|POS) = ??

Easy!

Not so easy!

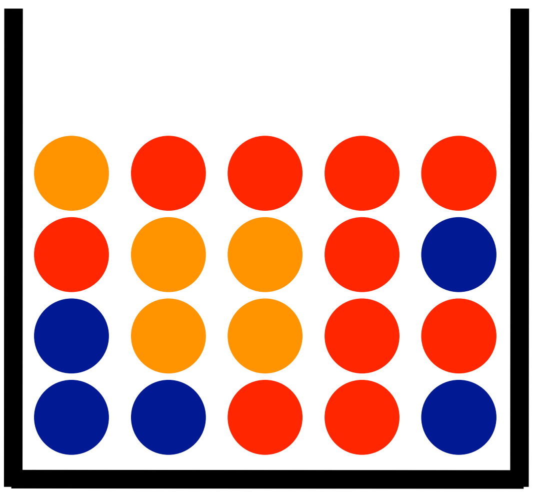
Naive Bayes Classification

example: positive/negative movie reviews

- Our next goal is to estimate these parameters from the training data!
- P(NEG) = % of training set documents that are NEG
- P(POS) = % of training set documents that are POS
- P(D|NEG) = ??
- P(D|POS) = ??

Remember Conditional Probability?

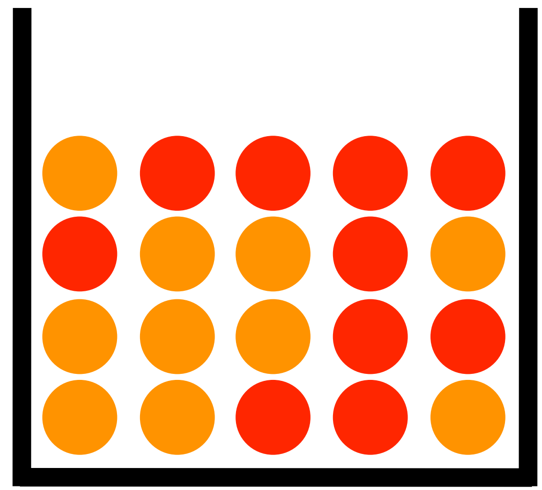
P(RED) = 0.50
P(BLUE) = 0.25
P(ORANGE) = 0.25



A

- P(● | A) = 0.25
- P(● | A) = 0.50
- P(● | A) = 0.25

P(RED) = 0.50
P(BLUE) = 0.00
P(ORANGE) = 0.50

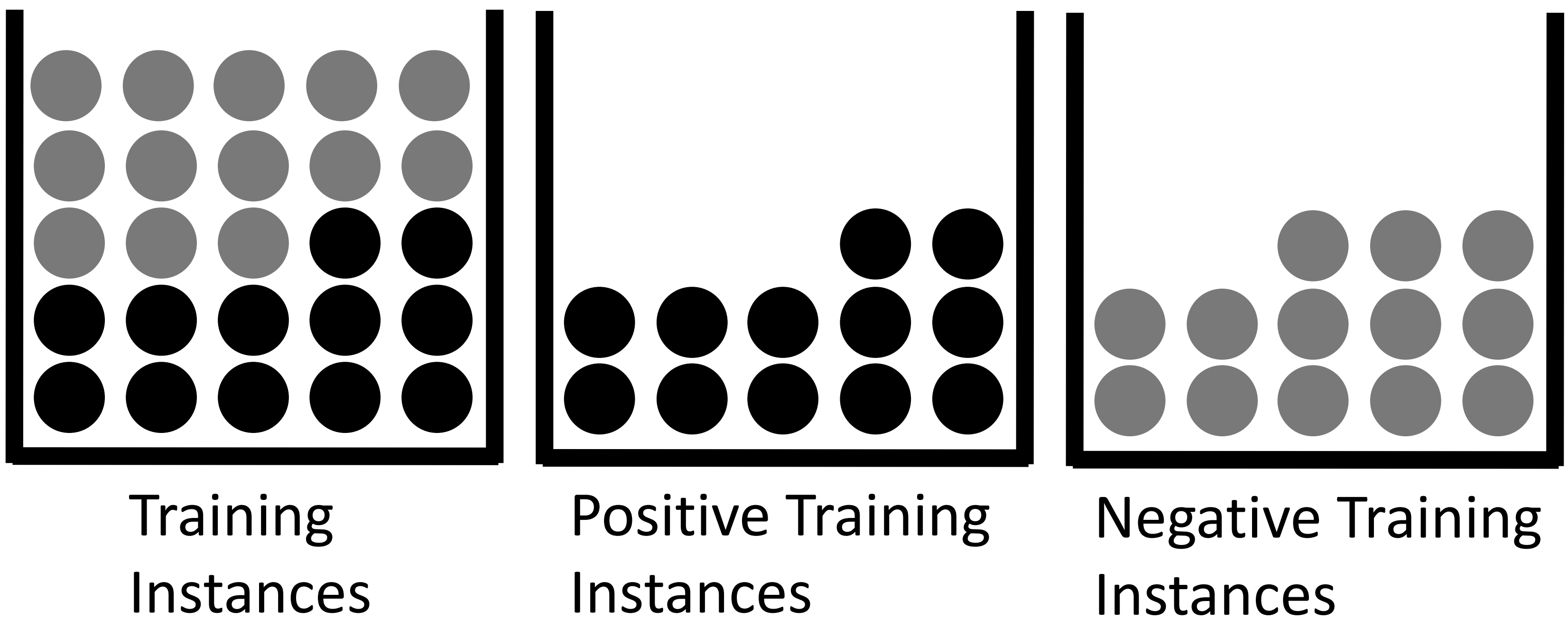


B

- P(● | B) = 0.00
- P(● | B) = 0.50
- P(● | B) = 0.50

Naive Bayes Classification

example: positive/negative movie reviews



P(D|POS) = ??

P(D|NEG) = ??

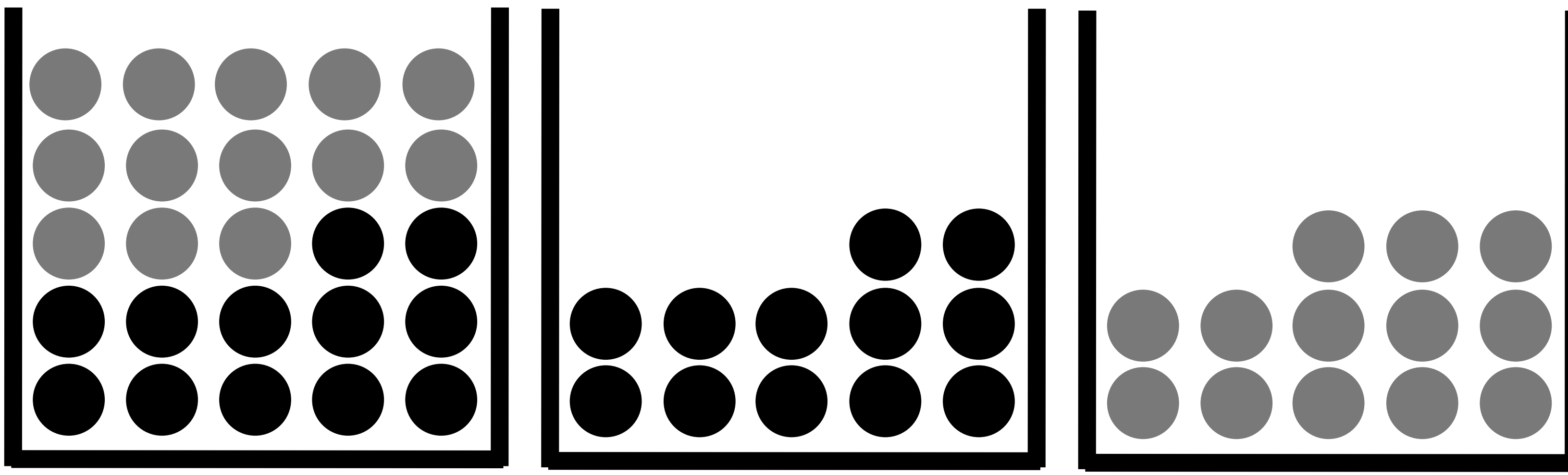
Naive Bayes Classification

example: positive/negative movie reviews

w_1	w_2	w_3	w_4	w_5	w_6	w_7	w_8	...	w_n	sentiment
1	0	1	0	1	0	0	1	...	0	positive
0	1	0	1	1	0	1	1	...	0	positive
0	1	0	1	1	0	1	0	...	0	positive
0	0	1	0	1	1	0	1	...	1	positive
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	...	⋮	⋮
1	1	0	1	1	0	0	1	...	1	positive

Naive Bayes Classification
example: positive/negative movie reviews

- We have a problem! What is it?



Training Instances

Positive Training Instances

$P(D|POS) = ??$

Negative Training Instances

$P(D|NEG) = ??$

Naive Bayes Classification

example: positive/negative movie reviews

- We have a problem! What is it?
- Assuming n binary features, the number of possible combinations is 2^n
- 2^{1000} = 1.071509e+301
- And in order to estimate the probability of each combination, we would require multiple occurrences of each combination in the training data!
- We could never have enough training data to reliably estimate P(D|NEG) or P(D|POS)!
- The document for the prediction in test data would not appear in training data

Naive Bayes Classification

example: positive/negative movie reviews

- Assumption: given a particular class value (i.e, POS or NEG), the value of a particular feature is independent of the value of other features
- In other words, the value of a particular feature is only dependent on the class value
- This is what Naïve means

Naive Bayes Classification

example: positive/negative movie reviews

w_1	w_2	w_3	w_4	w_5	w_6	w_7	w_8	...	w_n	sentiment
1	0	1	0	1	0	0	1	...	0	positive
0	1	0	1	1	0	1	1	...	0	positive
0	1	0	1	1	0	1	0	...	0	positive
0	0	1	0	1	1	0	1	...	1	positive
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	...	⋮	⋮
1	1	0	1	1	0	0	1	...	1	positive

Naive Bayes Classification

example: positive/negative movie reviews

- Assumption: given a particular class value (i.e, POS or NEG), the value of a particular feature is independent of the value of other features

- Example: we have <u>seven</u> features and D = **1011011**

- P(**1011011**|POS) =

  $P(w_1=\mathbf{1}|POS) \times P(w_2=\mathbf{0}|POS) \times P(w_3=\mathbf{1}|POS) \times P(w_4=\mathbf{1}|POS) \times P(w_5=\mathbf{0}|POS) \times P(w_6=\mathbf{1}|POS) \times P(w_7=\mathbf{1}|POS)$

- P(**1011011**|NEG) =

  $P(w_1=\mathbf{1}|NEG) \times P(w_2=\mathbf{0}|NEG) \times P(w_3=\mathbf{1}|NEG) \times P(w_4=\mathbf{1}|NEG) \times P(w_5=\mathbf{0}|NEG) \times P(w_6=\mathbf{1}|NEG) \times P(w_7=\mathbf{1}|NEG)$

Naive Bayes Classification

example: positive/negative movie reviews

- Question: How do we estimate $P(w_1=\mathbf{1}|POS)$?

Naive Bayes Classification

example: positive/negative movie reviews

w_1	w_2	w_3	w_4	w_5	w_6	w_7	w_8	…	w_n	sentiment
1	0	1	0	1	0	0	1	…	0	positive
0	1	0	1	1	0	1	1	…	0	negative
0	1	0	1	1	0	1	0	…	0	negative
0	0	1	0	1	1	0	1	…	1	positive
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	…	⋮	⋮
1	1	0	1	1	0	0	1	…	1	negative

Naive Bayes Classification

example: positive/negative movie reviews

- Question: How do we estimate $P(w_1=\mathbf{1}|POS)$?

	POS	NEG
$w_1 = \mathbf{1}$	a	b
$w_1 = \mathbf{0}$	c	d

$P(w_1=1|POS) = ??$

Naive Bayes Classification

example: positive/negative movie reviews

- Question: How do we estimate $P(w_1=\mathbf{1}|POS)$?

	POS	NEG
$w_1 = \mathbf{1}$	a	b
$w_1 = \mathbf{0}$	c	d

$$P(w_1=\mathbf{1}|POS) = a \,/\, (a + c)$$

Naive Bayes Classification

example: positive/negative movie reviews

- Question: How do we estimate $P(w_1=\mathbf{1/0}|POS/NEG)$?

	POS	NEG
$w_1 = \mathbf{1}$	a	b
$w_1 = \mathbf{0}$	c	d

$P(w_1=1|POS) = a / (a + c)$

$P(w_1=0|POS) = ??$

$P(w_1=1|NEG) = ??$

$P(w_1=0|NEG) = ??$

Naive Bayes Classification

example: positive/negative movie reviews

- Question: How do we estimate $P(w_2=\mathbf{1/0}|POS/NEG)$?

	POS	NEG
$w_2 = \mathbf{1}$	a	b
$w_2 = \mathbf{0}$	c	d

$\mathbf{P(w_2=1|POS) = a / (a + c)}$

$\mathbf{P(w_2=0|POS) = c / (a + c)}$

$\mathbf{P(w_2=1|NEG) = b / (b + d)}$

$\mathbf{P(w_2=0|NEG) = d / (b + d)}$

- The value of a, b, c, and d would be different for different features w_1, w_2, w_3, w_4, w_5, ... , w_n

Naive Bayes Classification

example: positive/negative movie reviews

- Given instance D, predict positive (POS) if:

$$P(D|POS) \times P(POS) \geq P(D|NEG) \times P(NEG)$$

- Otherwise, predict negative (NEG)

Naive Bayes Classification

example: positive/negative movie reviews

- Given instance D, predict positive (POS) if:

$$P(POS) \times \prod_{i=1}^{n} P(w_i = D_i|POS) \geq P(NEG) \times \prod_{i=1}^{n} P(w_i = D_i|NEG)$$

P(**1011011**|POS) P(**1011011**|NEG)

- Otherwise, predict negative (NEG)

Naive Bayes Classification

example: positive/negative movie reviews

- Given instance D = **1011011**, predict positive (POS) if:

$P(w_1=\mathbf{1}|POS) \times P(w_2=\mathbf{0}|POS) \times P(w_3=\mathbf{1}|POS) \times P(w_4=\mathbf{1}|POS) \times P(w_5=\mathbf{0}|POS) \times P(w_6=\mathbf{1}|POS) \times P(w_7=\mathbf{1}|POS) \times P(POS)$

$$\geq$$

$P(w_1=\mathbf{1}|NEG) \times P(w_2=\mathbf{0}|NEG) \times P(w_3=\mathbf{1}|NEG) \times P(w_4=\mathbf{1}|NEG) \times P(w_5=\mathbf{0}|NEG) \times P(w_6=\mathbf{1}|NEG) \times P(w_7=\mathbf{1}|NEG) \times P(NEG)$

- Otherwise, predict negative (NEG)

Naive Bayes Classification

example: positive/negative movie reviews

- We still have a problem! What is it?

Naive Bayes Classification

example: positive/negative movie reviews

- Given instance D = **1011011**, predict positive (POS) if:

$P(w_1=\mathbf{1}|POS) \times P(w_2=\mathbf{0}|POS) \times P(w_3=\mathbf{1}|POS) \times P(w_4=\mathbf{1}|POS) \times P(w_5=\mathbf{0}|POS) \times \mathbf{P(w_6=1|POS)} \times P(w_7=\mathbf{1}|POS) \times P(POS)$

$$\geq$$

$P(w_1=\mathbf{1}|NEG) \times P(w_2=\mathbf{0}|NEG) \times P(w_3=\mathbf{1}|NEG) \times P(w_4=\mathbf{1}|NEG) \times P(w_5=\mathbf{0}|NEG) \times P(w_6=\mathbf{1}|NEG) \times P(w_7=\mathbf{1}|NEG) \times P(NEG)$

- Otherwise, predict negative (NEG)

What if this never happens in the training data?

Smoothing Probability Estimates

- When estimating probabilities, we tend to ...
  - Over-estimate the probability of observed outcomes
  - Under-estimate the probability of unobserved outcomes
- The goal of smoothing is to ...
  - Decrease the probability of observed outcomes
  - Increase the probability of unobserved outcomes
- It’s usually a good idea
- You probably already know this concept!

Add-One Smoothing

- Question: How do we estimate $P(w_1 = \mathbf{1/0} \mid POS/NEG)$?

	POS	NEG
$w_1 = \mathbf{1}$	a	b
$w_1 = \mathbf{0}$	c	d

$P(w_1 = \mathbf{1} \mid POS) = a / (a + c)$

$P(w_1 = \mathbf{0} \mid POS) = c / (a + c)$

$P(w_1 = \mathbf{1} \mid NEG) = b / (b + d)$

$P(w_1 = \mathbf{0} \mid NEG) = d / (b + d)$

Add-One Smoothing

- Question: How do we estimate $P(w_1 = \mathbf{1/0} | POS/NEG)$?

	POS	NEG
$w_1 = \mathbf{1}$	a + 1	b + 1
$w_1 = \mathbf{0}$	c + 1	d + 1

$P(w_1=1|POS)$ = ??

$P(w_1=0|POS)$ = ??

$P(w_1=1|NEG)$ = ??

$P(w_1=0|NEG)$ = ??

Add-One Smoothing

- Question: How do we estimate $P(w_1=\mathbf{1/0}|POS/NEG)$?

	POS	NEG
$w_1 = \mathbf{1}$	a + 1	b + 1
$w_1 = \mathbf{0}$	c + 1	d + 1

$$P(w_1=1|POS) = (a + 1) / (a + c + 2)$$

$$P(w_1=0|POS) = (c + 1) / (a + c + 2)$$

$$P(w_1=1|NEG) = (b + 1) / (b + d + 2)$$

$$P(w_1=0|NEG) = (d + 1) / (b + d + 2)$$

Naive Bayes Classification

how to emphasize important features?

- Given instance D = **1011011**, predict positive (POS) if:

$P(w_1=\mathbf{1}|POS) \times P(w_2=\mathbf{0}|POS) \times P(w_3=\mathbf{1}|POS) \times P(w_4=\mathbf{1}|POS) \times P(w_5=\mathbf{0}|POS) \times P(w_6=\mathbf{1}|POS) \times P(w_7=\mathbf{1}|POS) \times P(POS)$

$$\geq$$

$P(w_1=\mathbf{1}|NEG) \times P(w_2=\mathbf{0}|NEG) \times P(w_3=\mathbf{1}|NEG) \times P(w_4=\mathbf{1}|NEG) \times P(w_5=\mathbf{0}|NEG) \times P(w_6=\mathbf{1}|NEG) \times P(w_7=\mathbf{1}|NEG) \times P(NEG)$

- Otherwise, predict negative (NEG)

What if this word seems to be really important in the training data?

Naive Bayes Classification

example: positive/negative movie reviews

- How can we emphasize important words (features)?
- Hint: it's very simple!
- Add the features redundantly!

Naive Bayes Classification

how to emphasize important features?

- Given instance D = **1011011**, predict positive (POS) if:

$$\frac{P(w_1=\mathbf{1}|POS) \times P(w_2=\mathbf{0}|POS) \times P(w_3=\mathbf{1}|POS) \times P(w_4=\mathbf{1}|POS) \times P(w_5=\mathbf{0}|POS) \times P(w_6=\mathbf{1}|POS) \times \underline{P(w_7=\mathbf{1}|POS) \times P(w_7=\mathbf{1}|POS) \times P(w_7=\mathbf{1}|POS) \times P(w_7=\mathbf{1}|POS) \times P(w_7=\mathbf{1}|POS)} \times P(POS)}{}$$

$$\geq$$

$$P(w_1=\mathbf{1}|NEG) \times P(w_2=\mathbf{0}|NEG) \times P(w_3=\mathbf{1}|NEG) \times P(w_4=\mathbf{1}|NEG) \times P(w_5=\mathbf{0}|NEG) \times P(w_6=\mathbf{1}|NEG) \times \underline{P(w_7=\mathbf{1}|NEG) \times P(w_7=\mathbf{1}|NEG) \times P(w_7=\mathbf{1}|NEG) \times P(w_7=\mathbf{1}|NEG) \times P(w_7=\mathbf{1}|NEG)} \times P(NEG)$$

W_7 are repeated five times.

- What does it mean?

Naive Bayes Classification

how to emphasize important features?

- If $P(w_7=\mathbf{1}|POS) = 0.4$ and $P(w_7=\mathbf{1}|NEG) = 0.1$

$P(w_1=\mathbf{1}|POS) \times P(w_2=\mathbf{0}|POS) \times P(w_3=\mathbf{1}|POS) \times P(w_4=\mathbf{1}|POS) \times P(w_5=\mathbf{0}|POS) \times P(w_6=\mathbf{1}|POS) \times \underline{P(w_7=\mathbf{1}|POS) \times P(w_7=\mathbf{1}|POS) \times P(w_7=\mathbf{1}|POS) \times P(w_7=\mathbf{1}|POS) \times P(w_7=\mathbf{1}|POS)} \times P(POS)$

0.01024

$$\geq$$

0.00001

$P(w_1=\mathbf{1}|NEG) \times P(w_2=\mathbf{0}|NEG) \times P(w_3=\mathbf{1}|NEG) \times P(w_4=\mathbf{1}|NEG) \times P(w_5=\mathbf{0}|NEG) \times P(w_6=\mathbf{1}|NEG) \times \underline{P(w_7=\mathbf{1}|NEG) \times P(w_7=\mathbf{1}|NEG) \times P(w_7=\mathbf{1}|NEG) \times P(w_7=\mathbf{1}|NEG) \times P(w_7=\mathbf{1}|NEG)} \times P(NEG)$

- $P(w_7=\mathbf{1}|POS)$ used to 4 times bigger than $P(w_7=\mathbf{1}|NEG)$, but it becomes about one thousand times bigger by being used five times.

Naive Bayes Classification

example: positive/negative movie reviews

- Given instance D, predict positive (POS) if:

$$P(POS) \times \prod_{i=1}^{n} P(w_i = D_i | POS) \geq P(NEG) \times \prod_{i=1}^{n} P(w_i = D_i | NEG)$$

- Otherwise, predict negative (NEG)

Naive Bayes Classification

- Naive Bayes Classifiers are simple, effective, robust, and very popular – “Simplicity-first”
- Assumes that feature values are conditionally independent given the target class value
- This assumption does not hold in natural language
- Even so, NB classifiers are very powerful
- It works very well particularly when combined with some of the attribute selection procedures
- Smoothing is necessary in order to avoid zero probabilities
- Important terms can be used redundantly

Any Questions?

Instance-Based Learning

Next Class